Energy Analysis of Viscoelasticity Effect in Pipe Fluid Transients

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This study investigates the energy relations and dissipations in viscoelastic pipeline under fluid transients. The investigation is carried out analytically using energy analysis and Fourier transform techniques for viscoelastic waterhammer governing equations. The analytical results show that the viscoelastic term in waterhammer models is wrongly referred to in literature as being wave damping/dissipation when in actual fact it is the work done by the fluid on the pipe and vice versa. The energy dissipation is actually occurred in the pipe-wall due to the viscoelastic material strain. Moreover, the energy transfer/exchange between the fluid and pipe-wall and energy dissipation in the pipe-wall due to viscoelasticity effect is relating to the ratio of the pipe viscoelastic frequency and the fluid wave frequency.
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Keywords: viscoelasticity, pipe system, fluid transients, energy analysis, wave dissipation

1 Introduction

The factors affecting pressure wave propagation and damping in real-world pipe systems include wall shear under unsteady turbulent conditions, material behavior of the pipe by viscoelastic effect, and energy transformations by interaction between fluids and pipes. Current waterhammer models cannot adequately represent the pressure wave dissipation observed in real-world pipe systems. To address this deficiency, researches in this field often attribute that wave dissipation in the governing models to either unsteady friction effects [1–3] or viscoelasticity effects [4–9].

Some recent developed techniques for the utilization of transients in the pipe fluid transients, such as leak detection and pipe defects monitoring, are mainly based on the information relating to the pressure head magnitude damping in time and/or frequency domains [10–16]. To what extent the damping/dissipation of pressure wave pulse is important to both the applications of the developed physical models and the practical utilizations. So far, these transients-based techniques could successfully provide good predictions for simple pipelines (e.g., single pipeline) in those references, but it still has a great of technical challenges for complex pipe systems (e.g., branched pipelines or network).

Furthermore, more and more viscoelastic pipes have been and will be used for practical engineering due to the easiness and feasibility of construction and maintenance, such as pipe materials with polyvinyl chloride (PVC), polyethylene (PE) and high-density polyethylene (HDPE). The contribution of pressure wave damping due to pipe material viscoelasticity makes those transients-based techniques in present forms impossible applied to the viscoelastic systems. Consequently, the understanding of the viscoelasticity effect on the pressure wave dissipation in pipe fluid transients is critical for the design and operation of those pipe systems and for the possible usage of viscoelastic pipeline systems to suppress pressure waves and for leak and defect detection method that are based on model damping analysis.

The objective of this study is to investigate the viscoelastic effect on the pressure wave dissipation/damping in pipe fluid transients through the energy analysis method. This paper studies the viscoelastic effect first from the energy relation aspects to understand the energy transfer/exchange between the fluid and pipe-wall during the pressure wave propagations. The analytical Fourier transform (FT) is then applied to the governing equations of the waterhammer and viscoelastic models to analyze the wave energy dissipation in the pipe-wall. The conclusions will be provided in the end.

2 Model Description

The governing equations of 1D viscoelastic waterhammer/transients can be expressed as (adapted from Refs. [6,8,9])

$$\frac{A}{\rho a^2}\frac{\partial P}{\partial t}+\frac{\partial Q}{\partial x}+2A\frac{\partial \varepsilon_r}{\partial t}=0 \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{A}{\rho}\frac{\partial P}{\partial x}+\frac{\pi D}{\rho}\tau_w=0 \tag{2}$$

where P is the piezometric pressure, ρ is the fluid density, x is the spatial coordinate, t is the temporal coordinate, g is the gravitational acceleration, Q is the discharge, D is the pipe diameter, A is the pipe cross-sectional area, a is the wave speed, ε_r is the strain of viscoelastic pipe-wall, and τ_w is the wall shear stress.

During the fluid transients, the two effects of viscoelasticity and unsteady friction on the pressure wave propagation are represented by the retarded term in Eq. (1) and the shear stress term in Eq. (2), respectively. From the hydraulic point of view, the retarded strain term is relating to the radial velocity of the pipe-wall due to the viscoelastic deformation, which is derived for the continuity Eq. (1) in Ref. [9] as

$$2A\frac{\partial \varepsilon_r}{\partial t}=q_R \tag{3}$$

in which q_R is the radial velocity of pipe-wall.

On the other hand, from the viscoelastic deformation mechanism of pipe materials, the retarded strain can be represented by the *Kelvin–Voigt* (K-V) model [6,9,17] as

$$C_0JP(x,t)=\tau\frac{\partial \varepsilon_r}{\partial t}+\varepsilon_r \tag{4}$$

in which C_0=coefficient, J and τ are the creep-compliance coefficient and relaxation time of the K-V model, respectively, and $P(x,t)$ is the instantaneous pressure in the system.

By considering the unsteady eddy viscosity effect according to different 1D unsteady friction models [18], the wall shear stress can be simplified as the following form:

$$\tau_w=\frac{4\rho\bar{\upsilon}_t}{DA}Q(t) \tag{5}$$

in which $\bar{\upsilon}_t$ is the scale of averaged turbulent eddy viscosity.

The form in Eq. (5) is just adopted herein to illustrate the importance of the unsteady friction from a scaling sense since the eddy viscosity during fluid transients is in actual time-dependent

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changing [19]. Furthermore in Ref. [20], it has been shown that the "frozen" viscosity model could provide good prediction in the pipe fluid transients.

3 Energy Analysis

By applying the following operations for governing equations, Eq. (1) and Eq. (2) [21],

$$\int_0^L \text{Eq.(1)} \times P dx \quad \text{and,} \quad \int_0^L \text{Eq.(2)} \times Q dx \tag{6}$$

gives

$$\frac{A}{2\rho a^2}\frac{\partial}{\partial t}\int_0^L P^2 dx + \int_0^L P dQ + 2A\int_0^L \left(\frac{\partial \varepsilon_r}{\partial t}P\right)dx = 0 \tag{7}$$

$$\frac{\rho}{2A}\frac{\partial}{\partial t}\int_0^L Q^2 dx + \int_0^L Q dP + \frac{4}{D}\int_0^L (\tau_w Q)dx = 0 \tag{8}$$

in which L is the pipe length.

According to Eq. (3), Eq. (7) becomes

$$\frac{A}{2\rho a^2}\frac{\partial}{\partial t}\int_0^L P^2 dx + \int_0^L P dQ + \int_0^L (q_R P)dx = 0 \tag{9}$$

Combining Eqs. (8) and (9) and considering the relationship

$$\int_0^L Q dP = [PQ]_0^L - \int_0^L P dQ \tag{10}$$

in which $[PQ]_0^L = [P(x,t)Q(x,t)]|_0^L = P(L,t)Q(L,t) - P(0,t)Q(0,t)$.

After the mathematical manipulations, this gives

$$\frac{dU}{dt} + \frac{dT}{dt} + D' + W_L' + W_R' = 0 \tag{11}$$

where $U(t) = (A/2\rho a^2)\int_0^L P^2(x,t)dx$ is the total internal energy in system, $T(t) = (\rho/2A)\int_0^L Q^2(x,t)dx$ is the total kinetic energy in system, $D'(t) = (4/D)\int_0^L \tau_w Q(x,t)dx$ is the rate of frictional dissipation, $W_L'(t) = [P(x,t)Q(x,t)]|_0^L$ is the rate of total work from two pipe-ends, and $W_R'(t) = \int_0^L P(x,t)q_R(x,t)dx$ is the rate of total work on pipe-wall.

Clearly in Eq. (11), the total energy rate is balanced among the internal energy, the kinetic energy, the frictional dissipation, and the total work from pipe-ends and pipe-wall. Specifically, the term of frictional dissipation rate, $D'(t)$, is expected to be always non-negative since from Eq. (5),

$$\tau_w Q(x,t) = \frac{4\rho \bar{\upsilon}_t}{DA}Q^2 \geq 0$$

However, the term of work rate from pipe-wall, $W_R'(t)$ may be positive or negative because the pipe-wall could be compressed or expanded during transient oscillations. Therefore, from this point of view, the viscoelastic term in waterhammer model is the work by the pressure force due to the interaction between fluid and pipe-wall.

To confirm this, a testing numerical experiment is conducted for a simple viscoelastic pipe system from Covas et al. [9]. This system contains a reservoir, a viscoelastic pipeline, and a valve at the end. The two waterhammer events are caused by the fast closure and opening of the end valve. By using the same parameters in Ref. [9], the dissipation rate from the unsteady friction and the work rate from pipe-wall are calculated and shown in Figs. 1(*a*) and 1(*b*) for the valve-closure and opening respectively.

It is clear from Figs. 1(*a*) and 1(*b*) that the work by the fluid on the pipe-wall (i.e., $WR'(t)$ in the pictures) is positive during fluid compression (i.e., when waterhammer wave is positive) and negative during fluid expansion (i.e., when waterhammer wave is

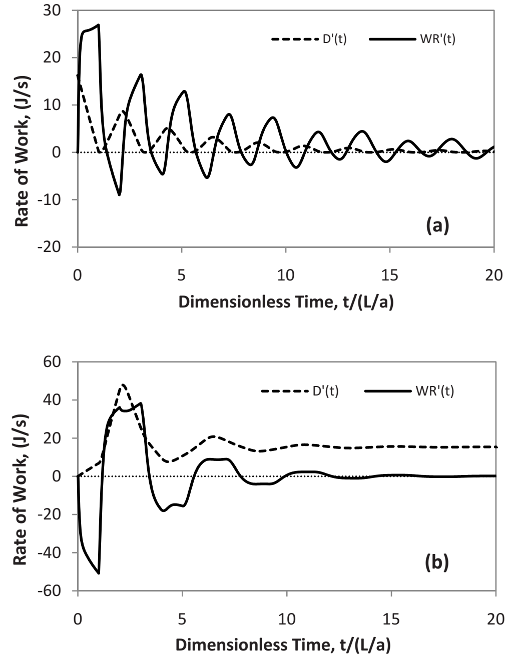

Fig. 1 Variation in energy rate due to friction and viscoelastic effects: (*a*) valve-closure and (*b*) valve opening

negative). That is, fluid does work on the pipe when waterhammer wave is positive and the pipe does work on the fluid when waterhammer wave is negative. Meanwhile, for the unsteady friction effect, the dissipation rate is always positive in the both cases from the pictures.

Moreover, it can be easily found that, during the whole hydraulic cycles, the total work from pipe-wall to fluid (positive) is not equal to the work from fluid to pipe-wall (negative). It is because there is energy dissipation in the pipe-wall after the energy absorbing/accumulating from the fluid. This dissipation will be discussed in the next section of this paper. So, what is thought of as viscoelastic dissipation term in waterhammer model is not dissipation, it is simply a matter of the work done by the fluid on the pipe is larger than the work done by the pipe on the fluid.

4 Fourier Analysis

By using the FT technique, the analytical solutions for the viscoelastic pipe fluid transients are provided in the attached Appendix. The derivations in Eqs. (A17) and (A20) show that energy dissipation due to unsteady friction effect and by pipe-wall viscoelastic effect are increasing with the wave frequency. In other words, the higher the pressures wave frequency, the more dissipation during the wave propagation. Furthermore, the result in Eq. (A22) shows that viscoelasticity effects are significant when $\tau < 1/\omega$ derivation. This is because the relaxation time scale τ of the viscoelastic pipe is the time needed for the pipe-wall to accommodate/absorb a part of the wave energy. That is, more energy absorption is expected if the wave period $T(=1/\omega)$ is larger than the viscoelastic relaxation time scale, and thus the more energy dissipation in the pipe-wall. As a result, the viscoelastic dissipation rate in the pipe-wall will be affected by both the pipe relaxation frequency (i.e., $1/\tau$) and the fluid wave frequency (i.e., $1/T$) or the ratio of the two quantities, i.e., T/τ.

As a result, the quantity of T/τ can be suggested as the roughly number of times the energy exchanged between fluid and pipe-wall. Clearly, the higher this number is the more energy transfer there is. It demonstrates that as time progresses in fluid transients, the viscoelasticity effect will become more and more remarkable and dominant due to this accumulated creep viscoelasticity effect.

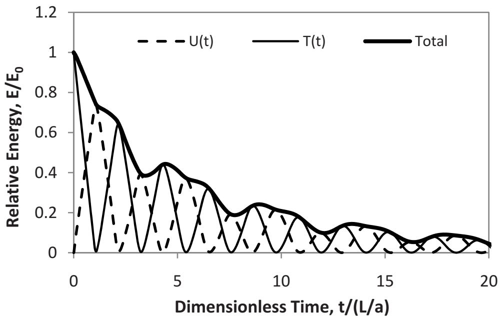

Fig. 2 Energy variation in pipe fluid transients caused by valve-closure

Indeed, this phenomenon could be easily found in the numerical and laboratory experimental tests, such as in Refs. [4–9].

Again, taking the aforementioned valve-closure case for example, the variations of different dimensionless energy forms (relative to the value of the initial state E_0) in the system are calculated and plotted in Fig. 2. By overall inspection, the total energy will be dissipated as expected due to the above mentioned unsteady turbulence and viscoelasticity effects. However, at some time points, the total energy will somewhat increase ("crest" of the total energy line in Fig. 2). This crest phenomenon can again be explained by the energy transfer between the fluid and pipe-wall. That is, the viscoelastic term in the model cannot only extract energy from fluid to pipe-wall but also get energy back from pipe-wall to the fluid (which is greater than the energy dissipated by the friction effect). Compared with the surrounding points, the crestbecomes more and more remarkable, which demonstrates again that the viscoelastic effect becomes relatively more pronounced and dominant as time progresses.

The amplitudes of the internal energy (i.e., $U(t)$ in Eq. (11)) and kinetic energy (i.e., $T(t)$ in Eq. (11)) in Fig. 2, which represent the pressure head and discharge quantities respectively, are also varying with the decreasing tendency on the whole. Furthermore, it clearly shows these two terms reciprocally transform to each other time and again. Therefore, the amplitude of the pressure wave oscillation will be damping when the total energy is dissipated gradually due to unsteady friction and viscoelastic effects.

From the former obtained results in terms of the energy dissipation, it can be concluded that the amplitude of the pressure wave will be deceasing with the increasing frequency and the number T/τ could be roughly the times of the damped quantity due to the viscoelastic effect in the pipe fluid transients.

Consequently, based on this analysis and results, it may provide guidelines for the design and operation of the viscoelastic pipelines in the practical industrial fluid system. For example, it could be useful for the design of the length of each viscoelastic pipe section in the system to avoid the probably encountered maximum/minimum pressure head. Meanwhile, it may suggest the additional energy supply for the viscoelastic transportation system since the viscoelastic dissipation under the transient state.

5 Conclusions

This paper investigates the effect of pipe viscoelasticity on the pressure wave dissipation in pipe fluid transients. The energy analysis of the viscoelastic waterhammer governing equation shows that the viscoelasticity term in the model just represents the work done by fluid on the pipe-wall or by pipe-wall on the fluid, which has been wrongly referred to in some literatures as being viscoelastic dissipation in additional to the turbulence dissipation. The viscoelastic dissipation is only occurred in the pipe-wall after absorbing/accumulating the energy from the fluid. Moreover, from the analytical analysis by Fourier transform, the energy transfer number between the fluid and pipe-wall is relating to the pipe viscoelastic creep and relaxation frequency and the fluid wave frequency, and thus, the damping of the pressure wave amplitude in the pipeline will increase with the ratio of these two frequencies (i.e., T/τ).

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Nomenclature

A = cross-sectional area
C_0, C_2, C_3, C_4, C_5 = coefficients
D = pipe diameter
$D'(t)$ = rate of frictional dissipation
E_0 = energy quantity of the initial state
$F(x,t)$ = function of Fourier transform
J = creep-compliance coefficient
K_{r-UF}, K_{r-VE} = real part of complex number
K_{i-UF}, K_{i-VE} = imaginary part of complex number
L = pipe length
P = piezometric pressure
Q = discharge
$T(t)$ = total kinetic energy
$U(t)$ = total internal energy
$W'_L(t)$, $W'_R(t)$ = rate of total work from two ends and from pipe-wall
a = wavespeed
b, c = function coefficients
g = gravitational acceleration
i = imaginary unit
q_R = radial velocity at pipe-wall
t = the temporal coordinate
x = the spatial coordinate

Greek Symbols

α = coefficient relating to boundary conditions
ε_r = retarded strain
λ = eigenvalues of partial equations
ν_t = kinetic, turbulent viscosity
ρ = fluid density
τ_w = wall shear stress
τ = relaxation time
ω = frequency

Appendix: Fourier Transform

At first, applying the following operation in waterhammer governing equations (1) and (2)

$$\frac{\partial}{\partial t}[\text{Eq.}(1)] - \frac{\partial}{\partial x}[\text{Eq.}(2)]$$

leads to

$$\frac{\partial^2 P}{\partial t^2} - a^2\frac{\partial^2 P}{\partial x^2} = -2\rho a^2\frac{\partial^2 \varepsilon}{\partial t^2} + \frac{4a^2}{D}\frac{\partial \tau_w}{\partial x} \tag{A1}$$

Similarly, applying the operation

$$a^2\left(\frac{\partial}{\partial x}[\text{Eq.}(1)]\right) - \frac{\partial}{\partial t}[\text{Eq.}(2)]$$

leads to:

$$\frac{\partial^2 Q}{\partial t^2} - a^2\frac{\partial^2 Q}{\partial x^2} = 2Aa^2\frac{\partial^2 \varepsilon}{\partial x\,\partial t} - \frac{\pi D}{\rho}\frac{\partial \tau_w}{\partial t} \tag{A2}$$

By introducing the Fourier transform and inverse Fourier transform between the time-domain and the frequency-domain for the quantity $F(x,t)$ at location x in the system [22]

$$\hat{F}(x,\omega)=\int_{-\infty}^{+\infty}e^{-i\omega t}F(x,t)dt \quad \text{and} \quad F(x,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{i\omega t}\hat{F}(x,\omega)d\omega \tag{A3}$$

The transform of Eqs. (4), (5), (A1), and (A2) will be

$$C_1J\hat{P}=(i\omega\tau+1)\hat{\varepsilon} \tag{A4}$$

$$\hat{\tau}_w \approx \frac{4\rho\bar{v}_t}{DA}\hat{Q} \tag{A5}$$

$$-\omega^2\hat{P}-a_0^2\frac{\partial^2\hat{P}}{\partial x^2}=2\rho\omega^2a_0^2\hat{\varepsilon}+\frac{4a_0^2}{D}\frac{\partial\hat{\tau}_w}{\partial x} \tag{A6}$$

$$-\omega^2\hat{Q}-a_0^2\frac{\partial^2\hat{Q}}{\partial x^2}=2Ai\omega a_0^2\frac{\partial\hat{\varepsilon}}{\partial x}-\frac{\pi D}{\rho}(i\omega)\hat{\tau}_w \tag{A7}$$

Combining the Eqs. (A4)–(A7) has

$$-\frac{\partial^2\hat{P}}{\partial x^2}=C_3\hat{P}-C_2\frac{\partial\hat{Q}}{\partial x} \tag{A8}$$

$$-\frac{\partial^2\hat{Q}}{\partial x^2}=C_5\hat{Q}-C_4\frac{\partial\hat{P}}{\partial x} \tag{A9}$$

in which $C_2=-16\rho\bar{v}_t/AD^2$, $C_3=(1+(2\rho a_0^2C_1J/1+i\omega\tau))(\omega^2/a_0^2)$, $C_4=-(2Ai\omega C_1J/1+i\omega\tau)$, and $C_5=(1-(4\pi\bar{v}_t/A\omega)i)(\omega^2/a_0^2)$.

By cancelling the terms relating to "Q" in Eqs. (A8) and (A9) gives

$$\frac{\partial^4\hat{P}}{\partial x^4}+(C_3+C_5-C_2C_4)\frac{\partial^2\hat{P}}{\partial x^2}+C_3C_5\hat{P}=0 \tag{A10}$$

That is

$$\frac{\partial^4\hat{P}}{\partial x^4}+b\frac{\partial^2\hat{P}}{\partial x^2}+c\hat{P}=0 \tag{A11}$$

in which $b=C_3+C_5-C_2C_4$ and $c=C_3C_5$.

The eigenvalues equation for Eq. (A11) is

$$\lambda^4+b\lambda^2+c=0 \tag{A12}$$

The general solution for λ is

$$\lambda^2=\frac{-b\pm\sqrt{b^2-4c}}{2} \quad \text{or} \quad \lambda=\pm\sqrt{\frac{-b\pm\sqrt{b^2-4c}}{2}}$$

Then, the solution of Eq. (A11) will be the following form [22]:

$$\hat{P}(x,\omega)=\alpha e^{\lambda x}=\alpha e^{K_rx}e^{iK_ix} \tag{A13}$$

in which α=coefficient relating to boundary conditions in system, $\lambda=K_r+K_ii$, K_r, and K_r are the factors relating to the dissipation rate and phase shift velocity of the pressure wave in frequency-domain.

Although it is difficult to obtain the general solution directly in Eqs. (A11) and (A12) due to the correlation term C_2C_4 the qualitative analysis based on some approximations and simplifications can be conducted for the unsteady friction and viscoelasticity, respectively, as follows.

(1) If neglecting the viscoelasticity effect, that is, only the unsteady friction effect in the system, i.e., $\tau=J=0$, gives

$$C_2=-\frac{16\rho\bar{v}_t}{AD^2}; \quad C_3=\frac{\omega^2}{a_0^2};$$

$$C_4=0 \quad \text{and} \quad C_5=\left(1-\frac{4\pi\bar{v}_t}{A\omega}i\right)\frac{\omega^2}{a_0^2}$$

The solution of Eq. (A12) is

$$\lambda_{1,2}=\pm\frac{\omega}{a}i; \quad \lambda_{3,4}=\pm(-K_{r-UF}+K_{i-UF}i) \tag{A14}$$

in which $K_{r-UF}=(\omega/a_0)\sqrt{(\sqrt{(A\omega)^2+(4\pi\bar{v}_t)^2}-A\omega)/2A\omega}$ and $K_{i-UF}=(\omega/a_0)\sqrt{(\sqrt{(A\omega)^2+(4\pi\bar{v}_t)^2}+A\omega)/2A\omega}$. Since it is expected that the unsteady friction can produce energy dissipation, the real solution of Eq. (A12) to this problem will be

$$\lambda=-K_{r-UF}+K_{i-UF}i \tag{A15}$$

and for the dissipation factor K_{r-UF} has

$$\frac{\partial K_{r-UF}}{\partial\bar{v}_t}>0 \tag{A16}$$

$$\frac{\partial K_{r-UF}}{\partial\omega}>0 \tag{A17}$$

(2) If neglecting the unsteady friction effect, that is, only considering the viscoelasticity effect, i.e., $k=0$, gives

$$C_2=0; \quad C_3=\left(1+\frac{2\rho a_0^2C_1J}{1+i\omega\tau}\right)\frac{\omega^2}{a_0^2};$$

$$C_4=-\frac{2Ai\omega C_1J}{1+i\omega\tau}; \quad \text{and} \quad C_5=\frac{\omega^2}{a_0^2}$$

The solution of Eq. (A12) is

$$\lambda_{1,2}=\pm(-K_{r-VE}+K_{i-VE}i); \quad \lambda_{3,4}=\pm\frac{\omega}{a}i \tag{A18}$$

in which

$$K_{r-VE}=\frac{\omega}{a_0}\sqrt{\frac{\sqrt{\Delta}-(1+\omega^2\tau^2+2\rho a_0^2C_1J)}{2(1+\omega^2\tau^2)}}$$

$$K_{i-VE}=\frac{\omega}{a_0}\sqrt{\frac{\sqrt{\Delta}+(1+\omega^2\tau^2+2\rho a_0^2C_1J)}{2(1+\omega^2\tau^2)}}$$

$$\Delta=(1+\omega^2\tau^2+2\rho a_0^2C_1J)^2+(2\rho a_0^2C_1J\omega\tau)^2$$

As a result, the solution for energy dissipation is

$$\lambda=-K_{r-VE}+K_{i-VE}i \tag{A19}$$

and for the dissipation factor K_{r-VE} has

$$\frac{\partial K_{r-VE}}{\partial\omega}>0 \tag{A20}$$

$$\frac{\partial K_{r-VE}}{\partial J}>0 \tag{A21}$$

$$\frac{\partial K_{r-VE}}{\partial\tau}<0 \quad \text{when} \quad \tau>\frac{1}{\omega}\sqrt{1+2\rho a^2C_0J}$$

$$\frac{\partial K_{r-VE}}{\partial\tau}>0 \quad \text{when} \quad \tau<\frac{1}{\omega} \tag{A22}$$

Especially, when $\tau\to 0(J\to 0)$ in Eq. (A18) gives

$$\lim_{\tau\to 0}K_{r-VE}=0$$

$$\lim_{\tau\to 0}K_{i-VE}=\frac{\omega}{a}\sqrt{1+2\rho a^2C_0J}\to\frac{\omega}{a}$$

These are the expected results in the elastic pipeline.

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